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Abstract

Black holes are among the most fascinating and remarkable objects in modern physics. Previously, they were considered only theoretical constructs written on paper, but now it is clear that they do exist and are located in the centers of nearly all galaxies. The strength of their gravitational field is so high that once light enters the zone called the event horizon, it cannot escape. This article will review some of the peculiarities of black holes. In particular, it will start with discussing the history of black holes and their formation as well as the mathematics of the escape velocity and the sizes of black holes. Advanced topics like bending of space-time by the black holes and their radiation will also be discussed.

1 Introduction

Black holes are regarded as most mysterious among all astronomical objects. They are considered regions in which gravity is so powerful that nothing can escape after crossing a certain boundary.

There is a misconception that a black hole is a hole in the space but it is not. Instead, it is an extremely dense concentration of mass that causes gravity to curve spacetime. According to the General Theory of Relativity by Albert Einstein, gravity is not a force rather it is the curvature of spacetime caused by mass and energy. That way a black hole is one of the remarkable predictions of the General Theory of Relativity.

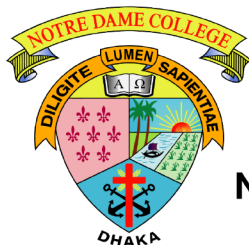
For years black holes were considered to be just a hypothesis. Now observations have proved their existence. The first detection of waves by the LIGO project in 2015 showed that black holes have the ability to merge by emitting huge amount of energy. In 2019 the Event Horizon Telescope managed to take the first-ever image of a shadow casted by a supermassive black hole, in M87 galaxy, providing one of the strongest pieces of evidence that supports General Relativity.

Therefore, black holes occupy a unique position in modern physics because they bring nearly every fundamental theory. Newtonian mechanics introduces the concept of escape velocity, General Relativity predicts their geometry, thermodynamics assigns them entropy and temperature, while quantum mechanics predicts Hawking radiation. Consequently, black holes provide one of the most promising pathways toward a unified description of nature.

This article aims to present the underlying physics of black holes in a mathematically accessible manner while emphasizing the physical ideas that make these objects one of the greatest scientific discoveries of the modern era.

2 A Brief History of Black Holes

The concept of the star whose gravitational field would not allow light to escape is far older than Einstein himself. In 1783, the English natural philosopher John Michell suggested the concept of



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dark stars. He believed that the velocity of light was much smaller than that of other bodies due to the nature of its particles and suggested that a star might become so massive and so dense that its escape velocity would exceed the speed of light. A few years later Pierre-Simon Laplace arrived at the same conclusion. However, because of the wave theory of light gaining more and more attention, this concept ceased to exist in science altogether.

In 1915, however, this topic gained momentum again due to the publication of the General Theory of Relativity by Albert Einstein. Einstein postulated that instead of being a force, the gravity is the consequence of the curvature of space-time caused by the presence of matter. Karl Schwarzschild found the first exact solution of the equation of field soon after that. His solution described the curved space-time around a spherically symmetric body.

Initially, many physicists including Albert Einstein doubted the very existence of such objects in nature.

In the 1930s, Subrahmanyan Chandrasekhar theoretically proved that a certain amount of mass of a star is required for it to be able to stay in a stable state indefinitely when its fuel is depleted. Afterward, J. Robert Oppenheimer and Hartland Snyder proved the possibility of forming black holes via gravitational collapse.

Then, the period of contemporary black-hole research started in the 1960s-1970s. Roger Penrose mathematically proved that gravitational collapse produces a singularity under general conditions, whereas Stephen Hawking united the ideas of quantum mechanics and General Relativity proving that black holes emit thermal radiation which later became known as Hawking radiation.

This way, the ideas described above have turned black holes from purely theoretical concepts into objects of fundamental astrophysical research.

3 The Birth of a Black Hole

Stars exist because two opposing processes remain in equilibrium. Gravity constantly pushes the star inward while nuclear fusion inside the core generates huge outward pressure. Hydrogen nuclei join together to become helium through fusion releasing energy that follows Einstein's well known equation,

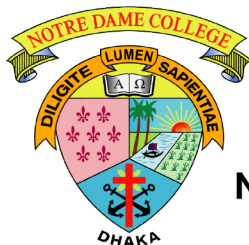
$$E = mc^2, \quad (1)$$

where m is the mass that turns into energy and c is the speed of light.

(1) As fusion continues heavier elements are made. Eventually iron builds up in the core. Because iron fusion uses energy of giving it the outward thermal pressure falls quickly.

When this support vanishes gravity takes over all forces. The stellar core collapses in a dramatic way, within just a few seconds causing a supernova explosion.

The final result depends mainly on the mass of the remaining core.



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- Low-mass stars become dwarfs.
- Intermediate-mass cores become neutron stars.
- Massive cores keep collapsing until an event horizon appears, creating a black hole.
- Low-mass stars become white dwarfs.
- Intermediate-mass cores become neutron stars.
- Extremely massive cores continue collapsing until an event horizon forms, producing a black hole.

4 The Mathematics Behind Black Holes

The concept of a black hole can be understood from just a simple idea in classical mechanics: *escape velocity*. Before Einstein's theory of General Relativity, Newtonian gravity already hinted that sufficiently compact objects could prevent light from escaping.

4.1 Newton's Law of Gravitation

According to Newton, the gravitational force between two masses is

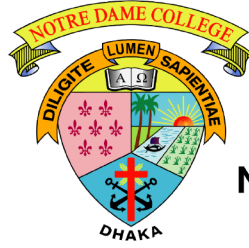
$$F = \frac{GMm}{r^2}, \quad (2)$$

where

- $G = 6.674 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ is the universal gravitational constant,
- M is the mass of the attracting body,
- m is the mass of the object,
- r is the distance between their centres.

The corresponding gravitational potential energy is

$$U = -\frac{GMm}{r}. \quad (3)$$



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4.2 Derivation of Escape Velocity

Suppose a particle of mass m is launched vertically upward from the surface of a spherical body of mass M and radius R .

To escape completely, the particle must reach infinity with zero remaining kinetic energy.

Initially,

$$E_i = \frac{1}{2}mv_e^2 - \frac{GMm}{R}. \quad (4)$$

At infinity,

$$E_f = 0. \quad (5)$$

Using conservation of mechanical energy,

$$E_i = E_f, \quad (6)$$

therefore,

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0. \quad (7)$$

Rearranging,

$$\frac{1}{2}mv_e^2 = \frac{GMm}{R}. \quad (8)$$

Cancelling the mass m ,

$$v_e = \sqrt{\frac{2GM}{R}}. \quad (9)$$

This equation shows that escape velocity depends only on the mass and radius of the celestial body.

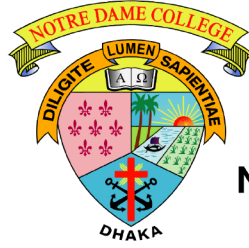
4.3 The Schwarzschild Radius

Now consider an extremely compact object.

If the escape velocity equals the speed of light,

$$v_e = c, \quad (10)$$

then substituting into the escape velocity equation gives



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$$c = \sqrt{\frac{2GM}{R}}. \quad (11)$$

Squaring both sides,

$$c^2 = \frac{2GM}{R}. \quad (12)$$

Hence,

$$R_s = \frac{2GM}{c^2}. \quad (13)$$

This critical radius is known as the **Schwarzschild radius**.

If an object's physical radius becomes smaller than R_s , no signal travelling at or below the speed of light can escape. The surrounding surface is called the **event horizon**.

(But we should keep in mind that the Schwarzschild radius is fundamentally a result of General Relativity, not Newtonian mechanics.)

4.4 Examples

For the Earth,

$$M_{\oplus} = 5.97 \times 10^{24} \text{ kg}, \quad (14)$$

which gives

$$R_s \approx 8.9 \times 10^{-3} \text{ m}, \quad (15)$$

or approximately

9 mm.

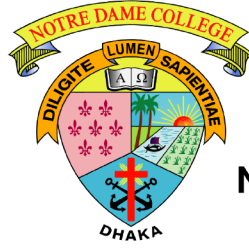
Thus, the Earth would become a black hole only if its entire mass were compressed into a sphere roughly the size of a marble.

For the Sun,

$$M_{\odot} = 1.989 \times 10^{30} \text{ kg}, \quad (16)$$

the Schwarzschild radius is

$$R_s \approx 2.95 \text{ km}. \quad (17)$$



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Since the Sun's actual radius is about

$$6.96 \times 10^5 \text{ km},$$

it is far too large to be a black hole in its present state.

5 Inside a Black Hole

As soon as the collapsing star enters the Schwarzschild radius, event horizon forms. This is not a real surface, instead it is a one-way boundary in the space-time continuum. A particle that crosses the event horizon will inevitably continue moving to the center, and there does not exist a future directed line that would allow it to escape from inside. An observer who sees an object that approaches the event horizon observes its gradual slowing down due to gravitational time dilation. However, for the falling observer, the crossing of the event horizon will take place in a finite proper time without reaching any special surface. According to General Relativity, a singularity should exist at the center of the non-rotating black hole, which means that the space-time curvature becomes formally infinite and the laws of physics break down. The presence of singularities implies that General Relativity is incomplete and should be somehow completed by quantum gravity.

6 Einstein's View: Curved Space-Time

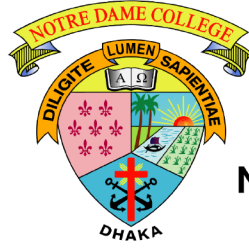
Newton's theory describes gravity as a force acting between two masses. Einstein revolutionized this picture by proposing that gravity is not a force at all. Instead, massive objects deform the geometry of spacetime, and freely moving bodies simply follow the straightest possible paths, called *geodesics*, in this curved geometry.

This revolutionary idea is summarized by Einstein's field equations,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \quad (18)$$

where

- $R_{\mu\nu}$ is the Ricci curvature tensor,
- R is the Ricci scalar,
- $g_{\mu\nu}$ is the metric tensor,
- $T_{\mu\nu}$ is the stress-energy tensor,



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- Λ is the cosmological constant.

John Archibald Wheeler beautifully summarized the meaning of this equation:

“Matter tells spacetime how to curve, and spacetime tells matter how to move.”

6.1 The Schwarzschild Metric

In 1916, Karl Schwarzschild obtained the first exact solution of Einstein’s field equations for a static, spherically symmetric mass.

The Schwarzschild metric is

$$ds^2 = - \left(1 - \frac{2GM}{rc^2} \right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2} \right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (19)$$

This equation completely describes the spacetime surrounding a non-rotating, electrically neutral black hole.

Notice that when

$$r = \frac{2GM}{c^2},$$

the coefficient of the metric becomes singular.

This radius is precisely the Schwarzschild radius derived earlier using Newtonian mechanics. Although the singularity at this radius is only a coordinate singularity, it marks the location of the event horizon.

7 Gravitational Time Dilation

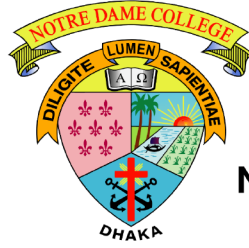
One of the most astonishing consequences of General Relativity is that time itself flows at different rates in different gravitational fields.

For a stationary observer outside a Schwarzschild black hole,

$$\Delta t = \frac{\Delta \tau}{\sqrt{1 - \frac{2GM}{rc^2}}} \quad (20)$$

where

- $\Delta \tau$ is the proper time measured by a nearby observer,



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- Δt is the time measured by a distant observer.

As r approaches the Schwarzschild radius,

$$r \rightarrow R_s,$$

the denominator approaches zero.

Hence,

$$\Delta t \rightarrow \infty.$$

To a distant observer, clocks near the event horizon appear to tick more and more slowly.

For the astronaut falling toward the black hole, however, nothing unusual happens at the event horizon itself. They cross it in a finite amount of proper time.

7.1 Gravitational Redshift

Light escaping from a strong gravitational field loses energy.

Its frequency decreases while its wavelength increases, producing gravitational redshift.

The relation is

$$f_{\infty} = f_r \sqrt{1 - \frac{2GM}{rc^2}} \quad (21)$$

where

- f_r is the emitted frequency,
- f_{∞} is the observed frequency far from the black hole.

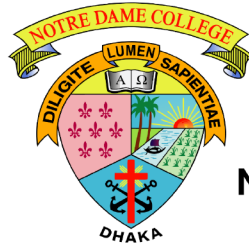
Near the event horizon,

$$f_{\infty} \rightarrow 0,$$

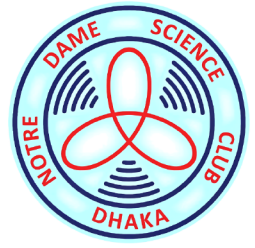
meaning the emitted light becomes infinitely redshifted.

8 Types of Black Holes

Current observations suggest four major categories of black holes.



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Stellar-Mass Black Holes

These form after the collapse of massive stars.

Typical masses are

$$3M_{\odot} \text{ to } 100M_{\odot}.$$

Intermediate-Mass Black Holes

These objects have masses between

$$10^2 M_{\odot} \text{ to } 10^5 M_{\odot}.$$

Only a handful of convincing candidates have been identified.

Supermassive Black Holes

These giants reside at the centres of galaxies.

Their masses range from

$$10^6 M_{\odot} \text{ to } 10^{10} M_{\odot}.$$

Sagittarius A*, located at the centre of the Milky Way, contains approximately four million solar masses.

Primordial Black Holes

These hypothetical black holes may have formed shortly after the Big Bang due to density fluctuations in the early Universe.

Although none has yet been confirmed, they remain candidates for explaining part of the dark matter content of the Universe.

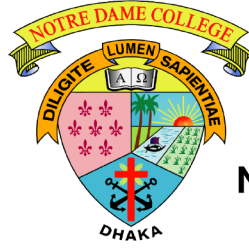
9 Accretion Disks and Relativistic Jets

Black holes themselves emit no light.

However, gas falling toward them forms a rapidly rotating **accretion disk**.

Friction within the disk heats the gas to temperatures exceeding millions of kelvin, producing intense X-rays.

Some rotating black holes also launch enormous **relativistic jets** extending thousands of light-years into space.



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These jets are believed to be powered by magnetic fields extracting rotational energy from the black hole through the Blandford–Znajek mechanism.

10 Hawking Radiation and Black Hole Thermodynamics

For decades, the black holes were considered perfect absorbers of everything passing through the event horizon. But this popular opinion was dramatically changed in 1974, when Stephen Hawking incorporated quantum mechanics and General Relativity and found out that black holes emit thermal radiation. According to quantum field theory, the empty space is not always empty. There are virtual pairs of particles and antiparticles which emerge and annihilate almost immediately. In the vicinity of the event horizon, one of the particles can fall into the black hole while another can escape to infinity. For an external observer, the particle that escapes to infinity looks like radiation emitted by the black hole. Since the black hole loses its energy, its mass will gradually decrease. As a result, black holes cannot be considered perfectly black; they evaporate very slowly.

10.1 Hawking Temperature

The temperature of a non-rotating Schwarzschild black hole is

$$T_H = \frac{\hbar c^3}{8\pi G M k_B} \quad (22)$$

where

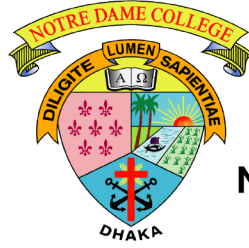
- $\hbar$ is the reduced Planck constant,
- k_B is the Boltzmann constant,
- G is the universal gravitational constant,
- M is the mass of the black hole.

This equation reveals an intriguing fact: the temperature is inversely proportional to the mass of the black hole.

Therefore,

$$T_H \propto \frac{1}{M}.$$

Massive black holes are colder than the cosmic microwave background, whereas extremely small black holes would be extraordinarily hot.



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10.2 Black Hole Entropy

Jacob Bekenstein proposed that black holes possess entropy because information disappears behind the event horizon.

The Bekenstein–Hawking entropy is

$$S = \frac{k_B A}{4l_P^2} \quad (23)$$

where

- A is the area of the event horizon,
- $l_P = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck length.

Unlike ordinary systems whose entropy depends on volume, black-hole entropy depends only on surface area.

This remarkable result inspired the holographic principle, suggesting that all information contained within a volume of space may be encoded on its boundary.

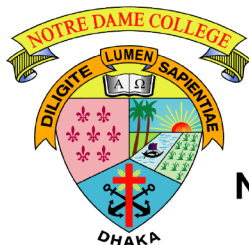
11 The Information Paradox

Quantum mechanics states that no information should ever be destroyed. However, Hawking radiation seems to be just a plain thermal radiation with absolutely no information about the matter from which the black hole was formed in the first place. Then what will happen to the information when a black hole totally disappears due to evaporation? This problem is referred to as Black Hole Information Paradox. Several solutions to the problem have been suggested. • The information is somehow transferred by quantum correlations in Hawking radiation. • Information is preserved in some microscopic leftovers. • The information is written on the surface of the black hole according to the holographic principle. • Quantum gravity may help us solve the problem once and for all.

12 Observational Evidence

Though black holes do not emit any radiation directly, there is plenty of evidence that they do exist.

In 2015 the Laser Interferometer Gravitational-Wave Observatory (LIGO) detected gravitational waves produced by the merger of two stellar-mass black holes.



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This observation provided the first direct confirmation of black-hole collisions and verified a major prediction of Einstein's General Relativity.

In 2019 the Event Horizon Telescope collaboration released the first image of the shadow of the supermassive black hole located at the centre of the galaxy M87.

In 2022 the collaboration also imaged Sagittarius A*, the supermassive black hole residing at the centre of our own Milky Way Galaxy.

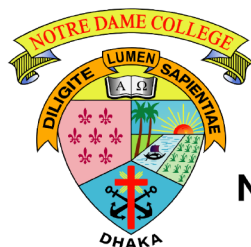
Together, these discoveries transformed black holes from theoretical predictions into directly observed astrophysical objects.

13 Conclusion

From Newton's theory of escape velocity and Einstein's spacetime curvature theory, black holes are clear examples of how gravity can become so powerful that it makes light unable to escape. Black holes have become laboratories for testing the theories of General Relativity, quantum mechanics, thermodynamics, and cosmology at the same time. As a result of studying black holes, many ideas were proposed, including Hawking radiation, entropy of a black hole, gravitational waves, and holographic principle. However, many basic questions are left unanswered yet. What really goes on at the singularity? How can gravity be reconciled with quantum mechanics? Can information survive black hole evaporation? Finding the answer to these questions could help in creating a unified theory of nature and thus black holes could turn out to be the brightest source of scientific discoveries.

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